

$$\tilde{y} = \gamma$$

$$D_m = \{(x_1, y_1), \dots, (x_n, y_n)\}$$

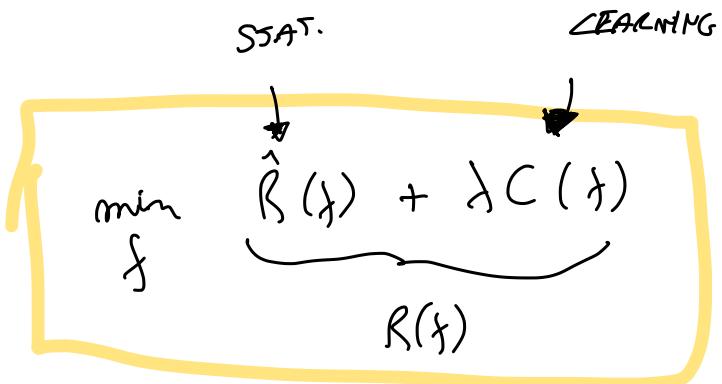
i. v. ol.

$$x \in \mathbb{R} \quad \gamma \in \mathbb{R}$$

$$f(x) = \sum_{i=0}^n c_i x^i$$

$$l(f(x), y) = (y - f(x))^2$$

$$C(\gamma) = \int S \left| \frac{d^i f}{dx^i} \right|^2 dx$$



$x \in \mathbb{R}^d$

$$= C_0$$

$$= " + C_1 x_1 + \dots + C_d x_d$$

$$= " + " + \bar{C}_1 x_1^2 + \dots + \bar{C}_d x_d^2 + \dots x_1 x_2 + \dots + x_{d-1} x_d$$

↑

↑

↑

$$\binom{d}{2} \approx d^2$$

$$\binom{d}{p} \approx d^p$$

$$d = 10^6$$

$$p = 6$$

$$\rightarrow (10^6)^6 \approx (10^6)^6 = 2^{100}$$

$$f(\underline{x}) = \underline{\omega} \cdot \underline{x}$$

$$\ell(f(x), y) = (y - f(x))^2$$

$$C(f) = \|\underline{\omega}\|^2$$

$$\begin{matrix} X \\ \in \mathbb{R}^{m \times d} \end{matrix} = \begin{bmatrix} \underline{x}_1' \\ \vdots \\ \underline{x}_m' \end{bmatrix} \quad y = \begin{bmatrix} ? \\ \vdots \\ ? \end{bmatrix}$$

$$\min_{\underline{\omega}} \sum_{i=1}^m (\underline{\omega} \cdot \underline{x}_i - y_i)^2 + \lambda \|\underline{\omega}\|^2$$

$$\min_{\underline{\omega}} \|\underline{X}\underline{\omega} - \underline{y}\|^2 + \lambda \|\underline{\omega}\|^2 \quad \text{RIDGE REGRESSION}$$

$$\nabla_{\underline{\omega}} (\underline{\omega}' \underline{X}' \underline{X} \underline{\omega} - 2 \underline{\omega}' \underline{X}' \underline{y} + \underline{y}' \underline{y} + \lambda \underline{\omega}' \mathbb{I} \underline{\omega}) = 0$$

$$2 \underline{X}' \underline{X} \underline{\omega} - 2 \underline{X}' \underline{y} + 2 \lambda \mathbb{I} \underline{\omega} = 0$$

$$(\underline{X}' \underline{X} + \lambda \mathbb{I}) \underline{\omega} = \underline{X}' \underline{y}$$

$$\begin{bmatrix} d \times d \\ \text{d.o.f.} \end{bmatrix} \begin{bmatrix} ? \\ \vdots \\ ? \end{bmatrix} = \begin{bmatrix} ? \\ \vdots \\ ? \end{bmatrix}$$

$$\min_{\underline{\omega}} \|\underline{X} \underline{\omega} - \underline{y}\|^2 + \lambda \|\underline{\omega}\|^2$$

$$A \begin{matrix} \underline{x} \\ \downarrow \end{matrix} = \underline{b}$$

$$\underline{\omega}^* = \underline{\omega}_\perp + \underline{\omega}_{||}$$

\times

$$\begin{aligned} \underline{\omega}_{||} &= \underline{X}' \underline{\alpha} \\ &= \sum_{i=1}^m 2x_i \underline{x}_i \end{aligned}$$

$$\underline{X} = \begin{bmatrix} \underline{x}_1' \\ \vdots \\ \vdots \\ \underline{x}_m' \end{bmatrix}$$

$$\|\underline{X}(\underline{\omega}_{||} + \underline{\omega}_\perp) - \underline{y}\|^2 + \lambda \|\underline{\omega}_{||} + \underline{\omega}_\perp\|^2$$

REP.
TH.

$$\|\underline{X}\underline{\omega}_{||} + \underbrace{\underline{X}\underline{\omega}_\perp}_{=0} - \underline{y}\|^2 + \lambda \|\underline{\omega}_{||}\|^2 + \cancel{\lambda \|\underline{\omega}_\perp\|^2}$$

$\underbrace{\quad}_{\geq 0} \quad \underbrace{\quad}_{\geq 0} \quad \underbrace{\quad}_{\geq 0} \quad \underline{\omega}_{||} ?$

$$\underline{\omega}^* = \underline{\omega}_{||}$$

$$\min_{\underline{x}} \| \underline{x} - \underline{y} \|^2 + \lambda \| \underline{x} \|^2$$

$$[\underset{d \times d}{\underbrace{\underline{x}\underline{x}'}}] [\underset{d}{\underline{y}}] = [\underset{d}{\underline{y}}]$$

$$\underline{x}^* = \underline{x}^1$$

$$\min_{\underline{Q}} \| \underline{x}\underline{x}' - \underline{y} \|^2 + \lambda \underline{\underline{Q}}' \underline{\underline{x}} \underline{x}' \underline{\underline{Q}} \rightarrow Q_{11} = \underline{x}'_1 \underline{x}_1 = Q_{11}$$

$$\underline{\underline{P}}_2 \left(\underline{\underline{Q}}' \underline{\underline{Q}} \underline{\underline{Q}} - 2 \underline{\underline{Q}}' \underline{\underline{y}} + \underline{\underline{y}}' \underline{\underline{y}} + \lambda \underline{\underline{Q}}' \underline{\underline{Q}} \right) = 0$$

$$\cancel{2\underline{\underline{Q}}' \underline{\underline{Q}} \underline{\underline{Q}} - 2 \cancel{\underline{\underline{Q}}' \underline{\underline{y}}} + \cancel{2 \lambda \underline{\underline{Q}}' \underline{\underline{Q}}}} = 0 \quad \underline{\underline{Q}} > 0$$

$$(\underline{\underline{Q}} + \lambda \underline{\underline{I}}) \underline{\underline{Q}} = \underline{\underline{y}}$$

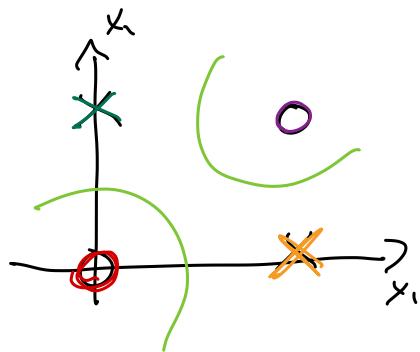
$\begin{matrix} \downarrow \\ \underline{x}\underline{x}' \\ n \times d \\ d \times m \\ m \times n \end{matrix}$

$$[\underset{n}{\underbrace{\underline{x}\underline{x}'}}] [\underset{n}{\underline{y}}] = [\underset{n}{\underline{y}}]$$

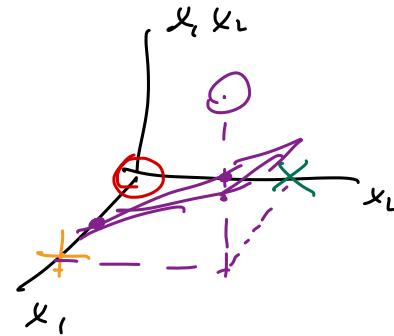
$$f(\underline{x}) = \underline{\psi}' \underline{x} = \underline{x}' Q \underline{x} = \sum_{i=1}^m \lambda_i \underbrace{\underline{x}_i' \underline{x}}_{\phi(\underline{x}_i)} \rightarrow \kappa(\underline{x}_i, \underline{x})$$

$$\underline{\lambda} : (Q + \lambda I) \underline{\lambda} = \underline{y}$$

$$\Rightarrow Q \underline{\lambda}_i = \underline{x}_i' \underline{x}_i \underbrace{\phi(\underline{x}_i)}_{\phi(\underline{x}_i)} \rightarrow \kappa(\underline{x}_i, \underline{x}_i)$$



$$\underline{\phi} = \begin{bmatrix} x_1 \\ x_1 \\ x_1 x_1 \end{bmatrix}$$



$$\underline{\psi} \underline{\phi}(\underline{x})$$

MERCER \rightarrow KEE NSCS $\mathbb{R}^d \times \mathbb{R}^{d^2} \rightarrow \mathbb{R}$

$$e^{-\gamma |x_i - x_j|} \quad , \quad x_i, x_j \in \mathbb{R}$$

Gaussian \rightarrow
 LB F

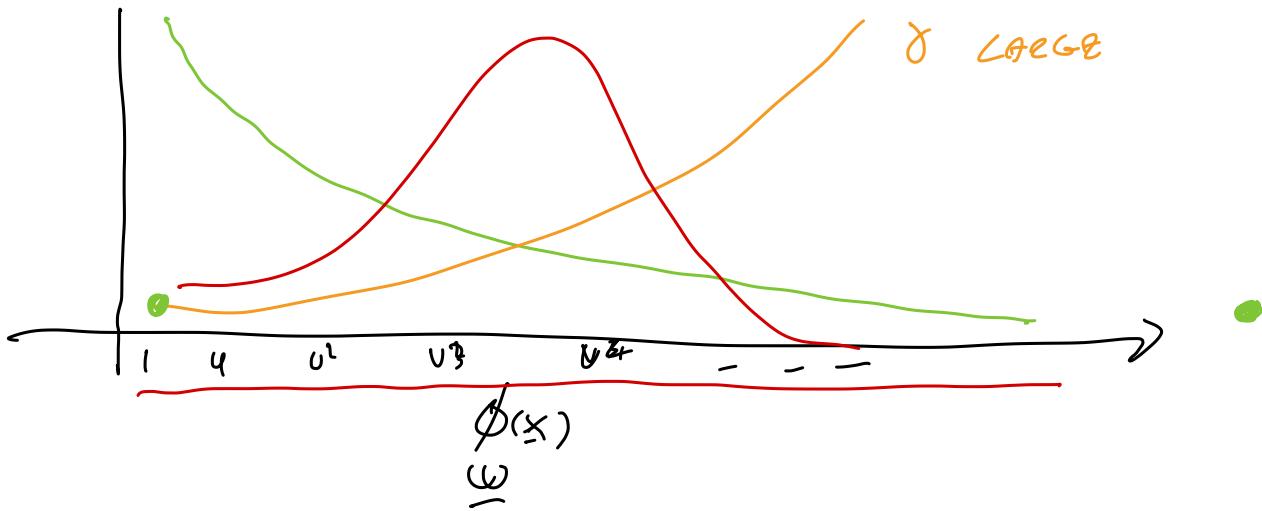
$\phi(x_i)$ $\phi(x_j)$

$$\begin{aligned}
 & e^{-\gamma(u-v)^2} \\
 &= e^{-\gamma u^2} e^{-\gamma v^2} e^{2\gamma uv} \\
 &= e^{-\gamma u^2} e^{-\gamma v^2} \sum_{i=0}^{\infty} \frac{1}{i!} (2\gamma uv)^i = \frac{-\partial u^i}{\partial u} \begin{bmatrix} 1 \\ \sqrt{\gamma} u^i \\ \sqrt{\gamma} u^i \end{bmatrix} \quad \frac{-\partial v^i}{\partial v} \begin{bmatrix} 1 \\ \sqrt{\gamma} v^i \\ \sqrt{\gamma} v^i \end{bmatrix} = \phi(u) \phi(v)
 \end{aligned}$$

$$\min_{\underline{w}} \|\underline{x} - \underline{y}\|^2 + \lambda \|\underline{w}\|^2$$

✓ small

✗ large



$$D_m = \{ (\underline{x}_1, \underline{y}_1), \dots, (\underline{x}_m, \underline{y}_m) \}$$

$$f(\underline{x}) = \underline{\phi}(\underline{x}) \quad K(\cdot, \cdot), \gamma$$

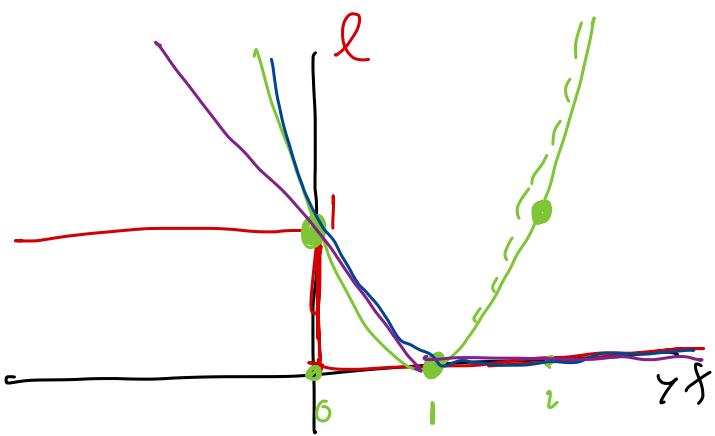
$$\ell(f(x_1), y) = (y - f(x_1))^2$$

$$C(f) = \| \underline{\phi} \|^2$$

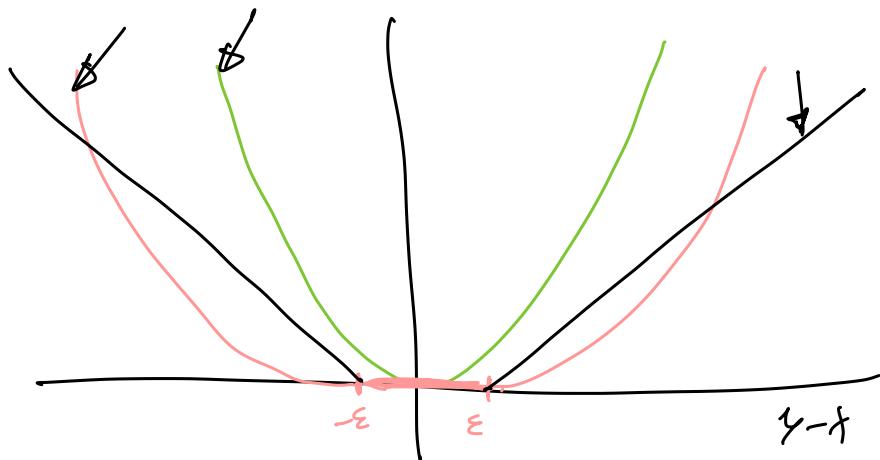
$$\min_{f} \hat{R}(f) + \lambda C(f)$$

CLASS.

$$y \in \{-1, 1\}$$

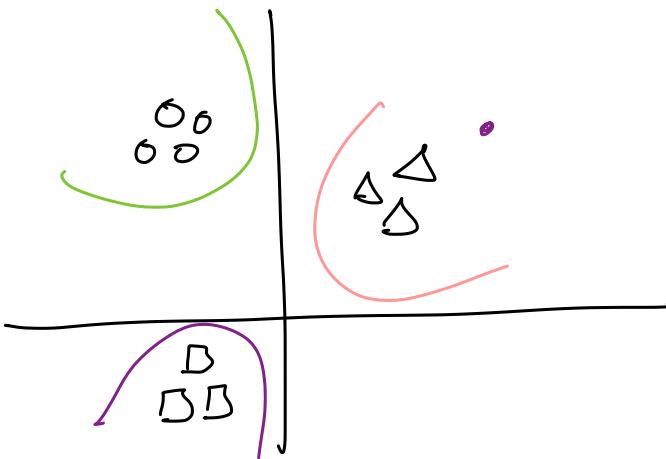


REGRESS



CGSS

$$Y = \{1, 2, 3, \dots, C\}$$



OVA

- C B.C. P.
- m samples
- CONFIDENCE
- UNBALANCED

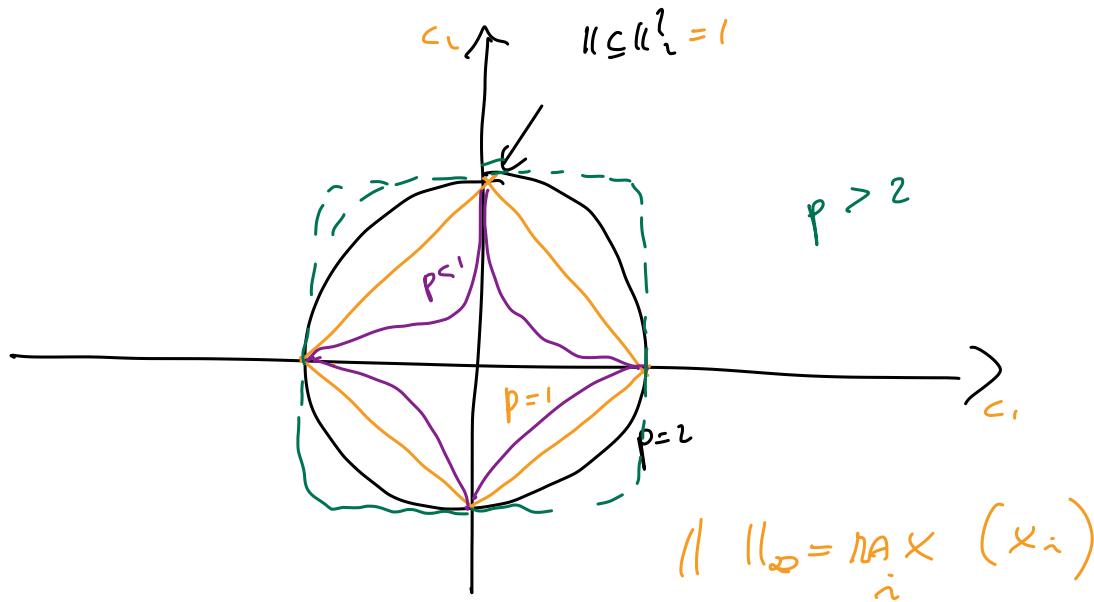
ARA

- $\binom{C}{2} \approx C^2$
- $2 \frac{m}{C}$
- $\pi \cdot V_0$
- BALANCED

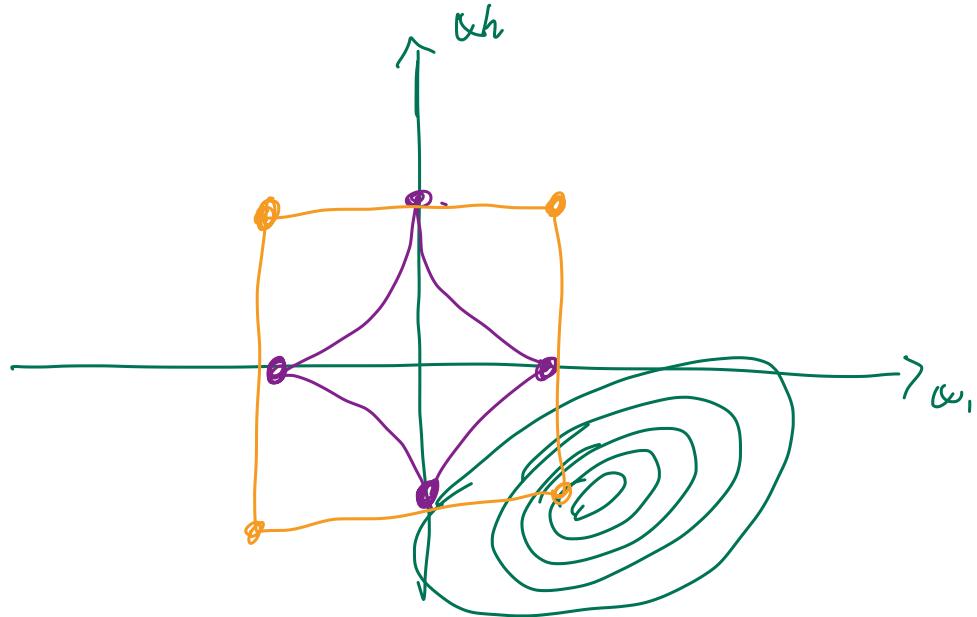
$$\|c\|_p = \|\underline{c}\|_p = \sqrt[p]{\sum_{i=1}^d |c_i|^{p_R}}$$

$$c_1 + c_2 = 1$$

$$|c_1| + |c_2| = 1$$



$$\min_{\underline{\omega}} \|\underline{\omega}\|^2 + \gamma \|\underline{\omega}\|_1$$



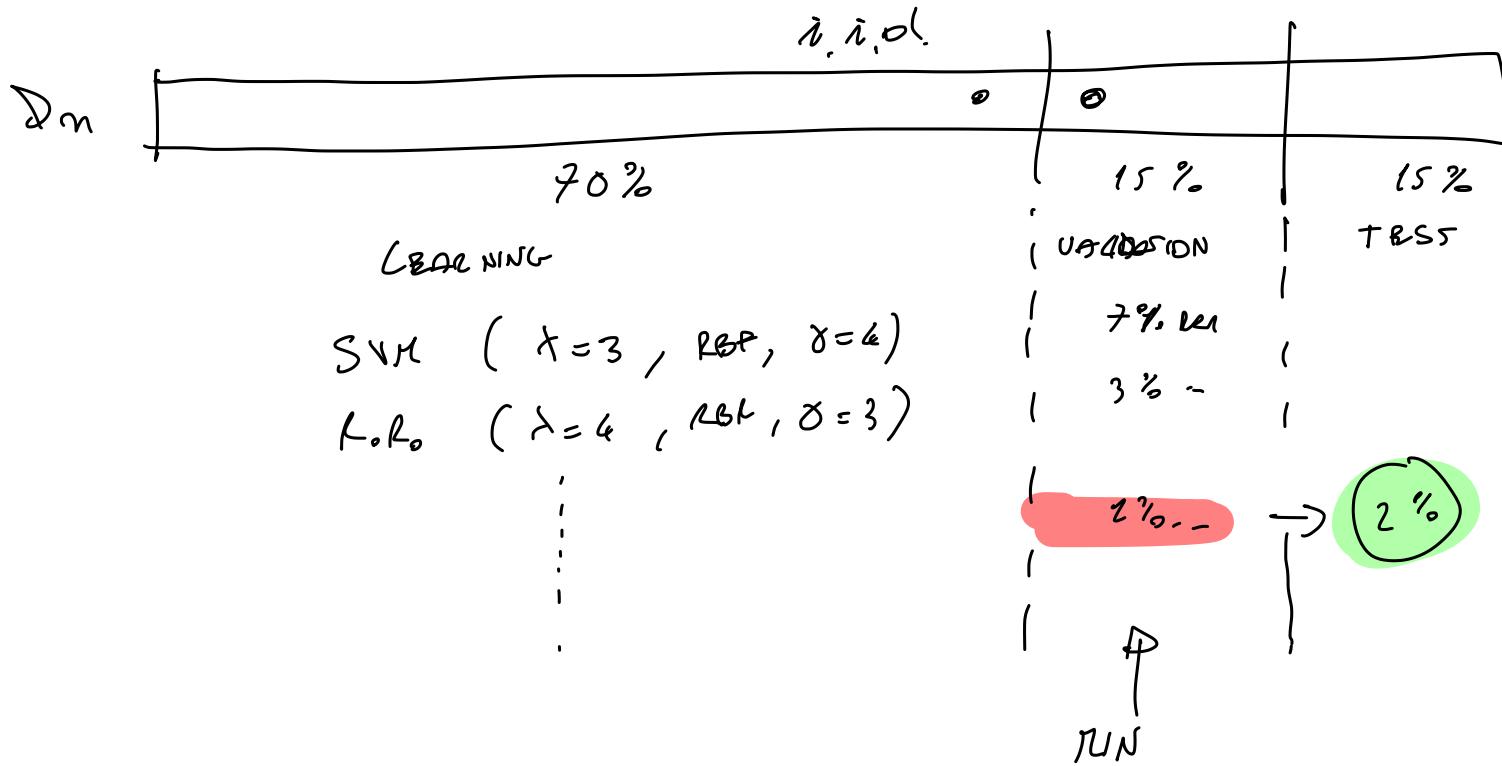
$$\mathcal{D}_n = \{(x_1, y_1), \dots, (x_n, y_n)\} \quad i.i.d.$$

$$f(\underline{x}) = \underline{\omega} \underline{\phi}(\underline{x})$$

$$l(f(\underline{x}), y) = \dots$$

$$C(f) = \dots$$

$$\min_f \hat{R}(f) + \lambda C(f)$$



MODEL SELECTION & ERROR BOUNDATION

WPSOR OF THE CROWDS

$$D_1 = 1\%$$

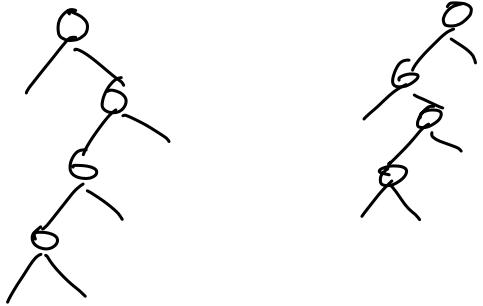
$$D_2 = 1\%$$

$$D_3 = 0,1\%$$

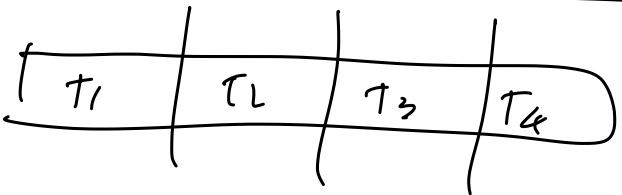
\approx INDRP.

D1	D2	D3	DL
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1 $\rightarrow 0,99 \cdot 0,01 = 0,0099$
0	0	0	0
1	0	0	1 $\rightarrow 0,01 \cdot 0,99 = 0,0099$
1	0	1	1 $\rightarrow 0,01 \cdot 0,01 = 0,0001$
1	1	0	1 $\rightarrow 0,01 \cdot 0,01 \cdot 0,999 = 0,000999$
1	1	1	1 $\rightarrow 0,01 \cdot 0,01 \cdot 0,001 \approx 0,0001 < 0,001$

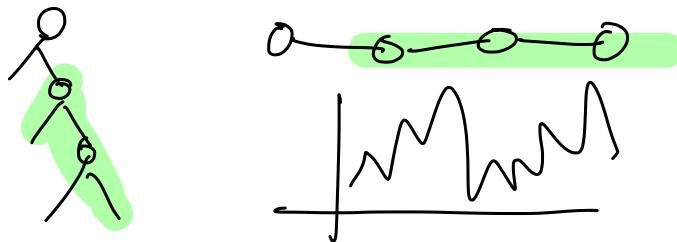
$$\begin{aligned} f(\underline{\omega}) &= \sum_i \omega_i x_i \\ &= \sum_{i=1}^d \omega_i x_i \\ &= \underline{\omega} \cdot \underline{x} \end{aligned}$$



R_F



$$D_m = \{x_i\}$$



$$x_1$$
$$x_2$$
$$x \rightarrow \phi(x) \in \mathbb{R}$$

Below the horizontal line, there are three diagrams. The first shows a red shaded irregular shape containing a stick figure, with a green arrow pointing to the label x_1 . The second shows a similar red shape containing a stick figure, with a green arrow pointing to the label x_2 . The third diagram shows a green shaded irregular shape with a stick figure inside, with a green arrow pointing to the expression $\phi(x) \in \mathbb{R}$. To the right of this expression is the mathematical formula $\binom{n}{p} = n^p$.

$$D_n = \{ (\bar{x}_i, y_i) \dots (\bar{x}_n, y_n) \} \quad \bar{x} \in \underset{\text{og}}{\mathbb{R}^d} \quad y \in \underset{\text{cross}}{\mathbb{C}^{2n}}$$

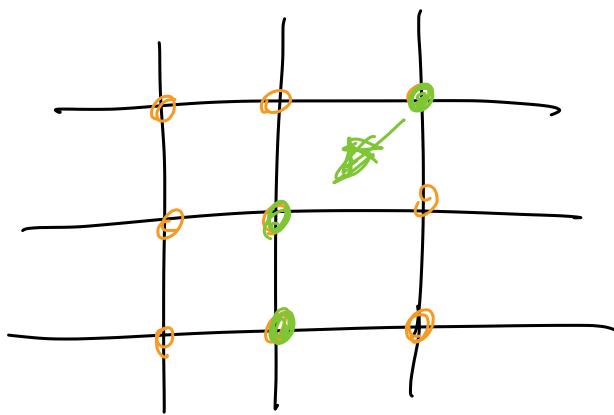
$$f(\underline{x}) = \dots$$

$$l(f(\underline{x}), y) = \dots$$

$$C(\delta) = \dots$$

$$\min_{\delta} \hat{R}(\delta) + \lambda C(\delta)$$

ReLU



($p \cdot m_p$)



Sob Servi

1060

